S15 C2

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

$$(a+b)^{10} = a^{10} + 10a^9b + 45a^8b^2$$

$$\left(2-\frac{x}{4}\right)^{10} = 2^{10} + 10 \times 2^9\left(-\frac{x}{4}\right) + 45 \times 2^8 \times \left(-\frac{x}{4}\right)^2$$

$$= 1024 - 1280x + 720x^2$$

2. A circle C with centre at the point $(2, -1)$ passes through the point A at $(4, -5)$.

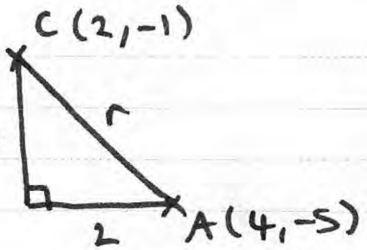
(a) Find an equation for the circle C.

(3)

(b) Find an equation of the tangent to the circle C at the point A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

a)



$$r^2 = 2^2 + 4^2 = 20$$

$$(x-2)^2 + (y+1)^2 = 20$$

b) $M_r = \frac{-4}{2} = \frac{-2}{1}$ $\quad$ M_t at $A = \frac{1}{2}$

$$y - y_1 = m(x - x_1) \Rightarrow y + 5 = \frac{1}{2}(x-4)$$

$$2y + 10 = x - 4 \Rightarrow x - 2y - 14 = 0$$

3. $f(x) = 6x^3 + 3x^2 + Ax + B$, where A and B are constants.

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 45,

(a) show that $B - A = 48$

(2)

Given also that $(2x + 1)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(4)

(c) Factorise $f(x)$ fully.

(3)

a) $f(-1) = 45 \Rightarrow -6 + 3 - A + B = 45 \Rightarrow -A + B = 48$

$\therefore B - A = 48$

b) $f(-\frac{1}{2}) = 0 \Rightarrow \cancel{-\frac{6}{8}} + \cancel{\frac{3}{4}} + -\frac{1}{2}A + B = 0 \quad \therefore \frac{1}{2}A = B$

$\underline{A = 2B}$

$\therefore \quad B - 2B = 48 \quad \therefore \quad B = -48$

$A = -96$

c) $f(x) = 6x^3 + 3x^2 - 96x - 48$

$\times$	$3x^2$	-48	
$2x$	$6x^3$	$-96x$	/
$+1$	$3x^2$	-48	/

$r = 0$

$(2x+1)(3x^2 - 48)$

$3(2x+1)(x^2 - 16)$

$= 3(2x+1)(x+4)(x-4)$

4.

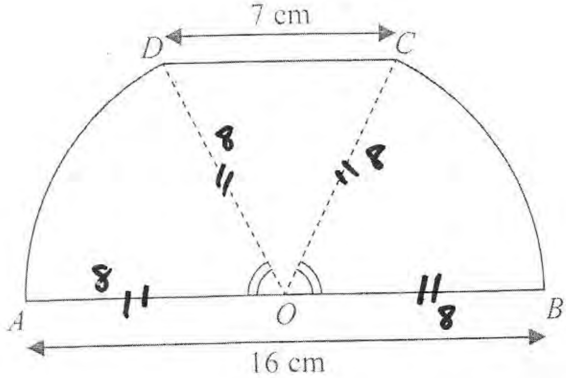


Figure 1

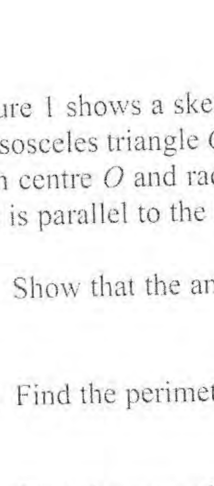
Figure 1 shows a sketch of a design for a scraper blade. The blade *AOBCDA* consists of an isosceles triangle *COD* joined along its equal sides to sectors *OBC* and *ODA* of a circle with centre *O* and radius 8 cm. Angles *AOD* and *BOC* are equal. *AOB* is a straight line and is parallel to the line *DC*. *DC* has length 7 cm.

(a) Show that the angle *COD* is 0.906 radians, correct to 3 significant figures. **(2)**

(b) Find the perimeter of *AOBCDA*, giving your answer to 3 significant figures. **(3)**

(c) Find the area of *AOBCDA*, giving your answer to 3 significant figures. **(3)**

a)



$\angle COD = 2 \times \sin^{-1}\left(\frac{3.5}{8}\right) = 0.905633^c$

$\therefore \angle COD \simeq 0.906^c$

b) Perimeter $= 16 + 7 + 2 \times \text{arc}AD$ $= r\theta$

$= 23 + 2 \times 8 \times \left(\frac{\pi - 0.906..}{2}\right)$

$= \underline{40.9}$ (3sf)

c) Area = triangle + Sector × 2

$= \frac{1}{2} \times 8 \times 8 \times \sin(0.906...) + 2 \times \frac{1}{2} \times 8^2 \times \left(\frac{\pi - 0.906}{2}\right)$

$= \underline{96.7}$

5. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

(a) the common ratio,

(4)

(b) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290

(4)

a) $U_1 = a$ $\quad S_2 = 34 = a + ar = a(1+r)$

$U_2 = ar$ $\quad \therefore a = \frac{34}{1+r}$

$S_\infty = \frac{a}{1-r} = 162$

$\Rightarrow a = 162(1-r)$

$162(1-r) = \frac{34}{1+r} \Rightarrow 1 - r^2 = \frac{34}{162} = \frac{17}{81}$

$\therefore r^2 = \frac{64}{81} \quad \therefore r = \frac{8}{9}$

b) $a = \frac{34}{1+\frac{8}{9}} = 18$

(ii) $a = 42 \quad r = \frac{6}{7} \quad S_n = 290 \quad S_n = \frac{a(1-r^n)}{1-r}$

$290 = \frac{42(1-(\frac{6}{7})^n)}{1-\frac{6}{7}} \Rightarrow 290 = 294(1-(\frac{6}{7})^n)$

$\frac{145}{147} = 1 - (\frac{6}{7})^n \quad \therefore (\frac{6}{7})^n = \frac{2}{147} \Rightarrow n = \log_{(\frac{6}{7})}(\frac{2}{147})$

$\Rightarrow n = 27.877..$

$\therefore$ Sum first exceeds 290 with 28th terms

6. (a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.

(4)

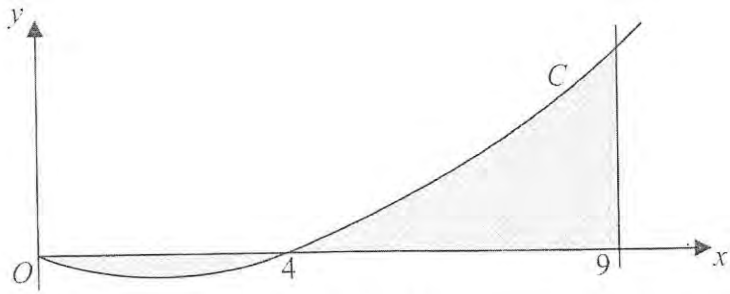


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve C starts at the origin and crosses the x-axis at the point $(4, 0)$.

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

a) $\int 10x^{\frac{3}{2}} - 20x \; dx = \dfrac{10x^{\frac{5}{2}}}{\frac{5}{2}} - \dfrac{20x^2}{2} + c$

$= 4x^{\frac{5}{2}} - 10x^2 + c$

b) $\left[4x^{\frac{5}{2}} - 10x^2\right]_0^4 = (-32) - (0) = -32$

$\left[4x^{\frac{5}{2}} - 10x^2\right]_4^9 = (162) - (-32) = 194$

$\therefore$ Area $= 194 + 32 = \underline{\underline{226}}$

7. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places. (3)

(ii) Find the values of y such that

$$\log_2(11y-3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

i) $2x+1 = \log_8 24 = 1.52832..$

$2x = 0.52832...$ $\quad x = \underline{0.264}$

2

ii) $\log_2(11y-3) - \log_2 3 - \log_2 y^2 = 1$

$\Rightarrow \log_2\left(\frac{11y-3}{3y^2}\right) = 1 \Rightarrow \frac{11y-3}{3y^2} = 2^1 = 2$

$\therefore 11y - 3 = 6y^2 \Rightarrow 6y^2 - 11y + 3 = 0$

$(3y-1)(2y-3) = 0$

$y = \frac{1}{3} \quad y = \frac{3}{2}$

8. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π. (3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k. (3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$ (3)

i) $\sin 3\theta = \sqrt{3}\cos 3\theta \Rightarrow \dfrac{\sin 3\theta}{\cos 3\theta} = \tan 3\theta = \sqrt{3}$

$\therefore 3\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$ etc

$\therefore \theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$

ii) $4(1-\cos^2 x) + \cos x = 4 - k$

$\cancel{4} - 4\cos^2 x + \cos x = \cancel{4} - k$

$\Rightarrow 4\cos^2 x - \cos x - k = 0$

$\Rightarrow \cos^2 x - \frac{1}{4}\cos x - \frac{1}{4}k = 0$

$\Rightarrow \left(\cos x - \frac{1}{8}\right)^2 - \frac{1}{64} = \frac{k}{4} = \frac{16k}{64}$

$\Rightarrow \cos x - \frac{1}{8} = \pm\sqrt{\frac{16k+1}{64}} \qquad \therefore \cos x = \dfrac{1 \pm \sqrt{16k+1}}{8}$

$k = 3 \Rightarrow \cos x = \dfrac{1 \pm 7}{8}$

$\cos x = 1 \Rightarrow \underline{x = 0}$

$\cos x = \frac{-6}{8} \Rightarrow x = \cos^{-1}\left(\frac{-3}{4}\right) = \underline{138.6}$ $\quad \underline{221.4}$

360 -

9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

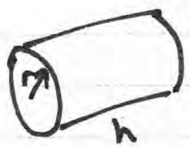
(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

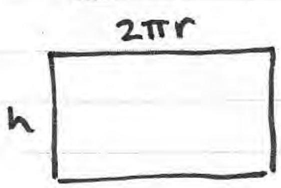
(1)

a) 

$V = \cancel{\pi} r^2 h = 75\cancel{\pi} \Rightarrow h = \frac{75}{r^2}$

top + bottom $= \pi r^2$ $\quad \therefore$ Cost $= 3\pi r^2$ $(\times 2)$ top + bottom $= 6\pi r^2$

$\times$ £3



$= 2\pi r h = \frac{2\pi r \times 75}{r^2} = \frac{150\pi}{r}$ $\quad \times$ £2

$\therefore$ Cost $= \frac{300\pi}{r}$

$\therefore$ total cost $= 6\pi r^2 + \frac{300\pi}{r}$ #

b) $C = 6\pi r^2 + (300\pi)r^{-1}$ $\quad$ at Min $C' = 0$

$C' = 12\pi r - (300\pi)r^{-2}$ $\quad 12\cancel{\pi}r = \frac{300\cancel{\pi}}{r^2}$

$C'' = 12\pi + 2(300\pi)r^{-3}$ $\quad = r^3 = \frac{300}{12}$

$\therefore r = 2.924...$

$\therefore$ min $C = 6\pi(2.924)^2 + \frac{300\pi}{2.924..}$

min $C =$ £483

c) $C'' = 12\pi + \frac{600\pi}{r^3}$ at $r = 2.924$ $\quad C'' = 36\pi > 0$ $\quad C'' > 0$ ∪

$\therefore$ MINIMUM